Theory of collision-dominated dust voids in plasmas

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A dust void, i.e., the dust-free region in a dusty plasma, results from the balance of the electrostatic and plasma (such as the ion drag) forces acting on a dust particle. The properties of dust voids depend on the ratio of the void size to the mean free path of plasma ions colliding with neutral species of a weakly ionized plasma. For many plasma-processing and plasma-crystal experiments, the size of the void is much larger than the ion-neutral mean free path. The theory and numerical results are presented for such a collisional case including the situations in which the plasma is quasineutral in the void region or the plasma quasineutrality is violated, as well as the case in which the ion ram pressure is insignificant.

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I. INTRODUCTION

Recent experiments [1–4] demonstrated that often the observed property of a dusty plasma is not a homogeneous distribution of dust particles but rather a formation of structured dust regions featuring also stable voids, i.e., dust-free regions inside the dust cloud. According to the proposed models [1,2,5] a dust void can appear in a low-temperature discharge plasma as a result of the balance of the electrostatic force and the drag force acting on dust particles, the latter due to the plasma ions flowing to the electrode. In general, the ion momentum balance in the void (i.e., in the dust-free region) is determined by the electric force, the ion pressure force, and the friction force of plasma ions with neutral species. When the size of the void is much less than the ion-neutral mean free path, the motion of plasma ions can be considered as collisionless. Thus the theory proposed earlier [5] for the case of collisionless plasma does not take into account the ion-neutral collision force. It was shown in [5] that a dust void indeed can be formed in a dusty plasma when the ion-neutral collisions are negligible and that the void has sharp boundaries where the dust density is sharply changed, but all other plasma parameters including the electron and ion densities are continuous. Such a ''collisionless'' void is globally stable for disturbances perpendicular to the void surface.

However, in many if not most of the existing experimental observations of dust voids [1–4], the size of the void is comparable to or even larger than the mean free path for the ion-neutral collisions and therefore the results of the theory [5] cannot be directly compared with them. Thus the theory of a collision-dominated dust void is necessary, which considers voids with sizes comparable to or larger than the ion-neutral mean free path. This theory should be based on the realistic model for the ion-neutral collisions fitting numerous experimental data already existing for low-temperature plasmas [6]. The other point that should be taken into account is relatively low ion flow velocities in the collision-dominated plasma, which can be of the order of the ion thermal velocity. Note that for the collisionless dust voids [5], the ion flow velocities much higher than the ion thermal velocity were considered. This limit is correct when the ion-neutral collisions are weak since the Mach number M of the ion flow at the void boundary was shown [5] to be close to unity. Note that even for near-sonic velocities of the flow $M\sim 1$, the ion drift speed is already $\sqrt{T_e/T_i}$ times larger than the ion thermal velocity [here, $T_{e(i)}$ is the electron (ion) temperature], and since in existing experiments $T_e \gg T_i$, the number $M=1$ corresponds to the ion flow velocities much (typically an order of magnitude) larger than the ion thermal velocity. If the ion-neutral collisions are sufficiently effective in the void region, they substantially lower the ion flow velocity, which can then become of the order of the ion thermal velocity [7]. The dust drug and dust capture coefficients [7,8] should take into account the fact that the ion drift velocity can be marginal as compared to the ion thermal velocity. Thus the corresponding generalization of the dust drug and capture rates should be taken into account.

These requirements are met in this paper presenting the theory of dust voids in a collision-dominated plasma. The theory is developed for the one-dimensional (1D) case, as in

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the model of Ref. [5]. The outline of the paper is the following. In Sec. II, we develop the model for the ion-neutral friction with the ion drift velocity dependence fitting the existing experimental data on the ion mobility in a low-temperature plasma. This expression can be used for the ion flow velocities comparable to or larger than the ion thermal velocity. In Sec. III, nonlinear equations in the void region are formulated for the case of a quasineutral plasma and for the case in which the plasma quasineutrality is violated (the quasineutrality in the void region can be maintained for the case in which the electron Debye length is less than the ion-neutral mean free path). Section IV is devoted to the solution of Poisson's equation in the dust region. This solution is derived taking into account the ion-neutral collisions and arbitrary dust drift velocities. It gives the parameters of the void boundary from the dust side (the details of the solution are given in the Appendix). Section V presents numerical solutions of the nonlinear equations for the void and the void boundary, allowing us to find the parameter range in which the void can exist, and it gives explicit results for the dependence of the size of the voids on the ionization rate and the ion friction force coefficients. The results are presented for conditions in which the quasineutrality is maintained both in the void and in the dust region and compared with those when the quasineutrality is violated. The results taking into account the ion ram pressure are also presented; they can be used to elucidate the role of the ion-neutral collisions in the collisionless void and to show the continuous transition of the collisionless dust void to the collision-dominated dust void. In the Appendix, we also give the exact expressions for the dependences of the dust drag and dust capture coefficients as functions of the ion-drift velocities valid for any relation between the ion drift velocity and the ion thermal velocity. These expressions containing the known error functions are useful for numerical computations.

II. THE MODEL FOR THE ION-NEUTRAL COLLISIONS AND THE ION MOBILITY

First, we introduce dimensionless quantities relevant to the high ion-neutral collision rates and emphasize physical grounds for such a normalization. The ion density N_i, the electron density N_e, and the electrostatic potential φ are normalized similarly to the collisionless case Ref. [5], i.e., the densities are in units of the ion density N_{i0} in the center of the void: $n=N_i/N_{i0}$, $n_e=N_e/N_{i0}$, and the potential is normalized to the electron temperature: $\phi=e\varphi/T_e$, where e is the electron charge and the electron temperature T_e is measured in energy units (i.e., the Boltzmann constant is unity). The (dimensional) electric field $\mathcal{E}$ and the distance X are normalized as $E=\mathcal{E}ed_i^2/a_dT_e$ and $x=Xa_d/d_i^2$, where $d_i=(T_i/4\pi N_{i0}e^2)^{1/2}$ is the ion Debye length, T_i is the ion temperature, and a_d is the radius of a dust particle. Note that in the case in which the ion-neutral collisions are neglected [5], only the mean free paths for the collisions of plasma particles with dust are important. For the collision-dominated case of interest, there appears here a new charactersitic length related to the ion-neutral mean free path. Since all the lengths (i.e., including those related to the mean free paths) are normalized to the same distance d_i^2/a_d, the dimensionless electric field is measured in units of the field in which an electron receives the energy T_e on the distance equal to the plasma particle mean free path in their collisions with dust particles. The density of the latter is related to the parameter $P\equiv N_dZ_d/N_{i0}$. Note that in general the plasma particle mean free path for their collisions with dust is d_i^2/a_dP, which is obviously d_i^2/a_d for $P=1$. The velocity V_i of the ion flow is normalized to $\sqrt{2}$ times the ion thermal velocity: $u=V_i/\sqrt{2}V_{Ti}$, where $V_{Ti}=(T_i/m_i)^{1/2}$ and m_i is the ion mass.

The number density of plasma neutral species is usually much higher than the ion number density (i.e., the plasma has a low degree of ionization) and the change in the neutral distribution due to the ion-neutral collisions can be neglected. The ion-neutral collisional friction slows down the ion flux and for low ion flux velocities the friction force is proportional to the ion drift velocity. Thus for the case in which only the electric field and the friction with neutral species are important, the total dimensionless force acting on plasma ions can be written as

$$F=-\frac{2u}{x_n}+E. \tag{1}$$

The factor 2 in the definition of the friction force is introduced for further convenience. The force F is written in the dimensionless units introduced above and thus $x_n/2$ is the ion mobility in these units: for $F=0$ we have $u=(x_n/2)E$. We use for the mobility the notation $x_n/2$ since in our dimensionless units x_n is the ion mean free path for the ion collisions with neutral species. The values for the ion mobility are well known from the experimental data in a low-temperature plasma [6]; thus the introduced dimensionless mobility $x_n/2$ can be easily calculated. The experimental data also show that with the increase of the electric field E, the mobility starts to depend on E and for large field $u\propto\sqrt{E}$. The interpolation between these two limits can in principle be also used. However, in general the use of the concept of mobility, which is appropriate if we do take into account only the friction and electric field force, is restricted. As soon as we include other forces acting on plasma ions, the balance of electric and friction force is violated. We need therefore to obtain an expression for the friction force dependence on the ion drift velocity, which leads in the presence of only the balance of the electric and friction forces to the mentioned changed in the dependence of the mobility on the electric field. Such an expression for the friction force F_u can be written as

$$F_u=-\frac{u}{x_n}(2+\alpha_u u). \tag{2}$$

The balance of the electric and friction forces then leads to

$$u=\frac{1}{\alpha_u}(\sqrt{1+x_n\alpha_u E}-1), \tag{3}$$

which gives us both the limits ($x_n\alpha_u E\ll 1$ and $x_n\alpha_u E\gg 1$) with the known dependence of the mobility on the electric field. The interpolation can then be used to obtain the numerical values of the coefficient α_u for the known experimental data. But even without calculations, we can conclude that in our dimensionless units the parameter α_u should be of order unity. Indeed, if the cross section σ does not strongly depend on the ion velocity, the friction force can be estimated as $\langle n v \sigma \rangle u$, giving $\langle n\sigma\rangle u$ for $u \ll 1$ (when averaging, we assume v to be of the order of the ion thermal velocity) and $\langle n\sigma\rangle u^2$ for $u \gg 1$. The fact that we use $\alpha_u \approx 1$ in the numerical computations is also discussed in the subsequent section.

If we need to consider other forces acting on plasma ions, we can use expression (2) together with these forces in the ion balance equation. If a void is in the intermediate regime where the ion-neutral collisions (although being important) do not dominate the ion ram pressure forces, we should perform calculations taking into account the friction, the ram pressure, and the electric-field forces. In this case, expression (3) is violated and the ion balance equation appears as an additional equation.

In the description of a void, it is necessary to know the dependence of forces acting on the dust particles (such as the drag and the ion capture force) on the ion drift velocity when the latter is close to the ion thermal velocity. These coefficients can be calculated in the standard way [8] (since the cross sections are well known) and they are presented in the Appendix for an arbitrary value of the ratio of ion drift velocity to the ion thermal velocity. Thus all forces can be written as functions of the ion drift velocity, and the case most interesting for applications (where the ion drift velocity is of the order of the ion thermal velocity) can be theoretically investigated.

III. EQUATIONS IN THE VOID REGION

The electron drift velocity in the void region cannot be larger than the ion flow velocity, i.e., it is much less than the electron thermal velocity, and thus the electron friction with neutral species is much less than the electric and the electron pressure force. This leads to the following equation (and as a consequence to the Boltzmann distribution for electrons):

$$\frac{1}{n_e}\frac{dn_e}{dx} = -E = -\frac{d\psi}{dx}, \qquad n_e = \exp(-\psi), \tag{4}$$

where $\psi = -\phi$ is the (dimensionless) minus electrostatic potential. We introduced the potential ψ in this way to have it positive in the void and to see explicitly that the number of electrons decreases as the potential increases when we reach the boundary of the void.

According to the known dependence of the ionization rate for the electron impact ionization, we assume that the ionization rate is proportional to the electron density. We then introduce x_i, the length (in the units used) at which the ionization causes the electron density to become unity. By denoting the ion flux by $\Phi = nu$ and by taking into account the ion ram pressure together with the electric field and the ion-neutral collision forces acting on ions, we obtain the system of equations in the void region, as follows.

(i) The Poisson equation,

$$\frac{dE}{dx} = -\frac{\tau}{a^2}\left[\exp(-\psi) - \frac{\Phi}{u}\right]; \tag{5}$$

(ii) the ion momentum balance equation (which in general contains on the left-hand side the ion ram pressure and the ion thermal pressure and converts to the balance of electric and neutral friction force if the ram pressure and ion thermal pressure are neglected),

$$\tau\left(\frac{du^2}{dx} + \frac{1}{n}\frac{dn}{dx}\right) = -\frac{u}{x_n}(2 + \alpha_u u) + E; \tag{6}$$

(iii) the expression of the (minus) electrostatic potential via the electric field,

$$\frac{d\psi}{dx} = E; \tag{7}$$

(iv) the ion continuity equation with the ionization term on the right-hand side,

$$\frac{d\Phi}{dx} = \frac{\exp(-\psi)}{x_i}. \tag{8}$$

Here, the dimensionless quantities τ and a are given by

$$\tau = \frac{T_i}{T_e}, \qquad a = \frac{a_d}{d_i}. \tag{9}$$

This system of four equations contains four variables n, u, ψ, and E. We can further simplify it using some extra assumptions.

The ion ram and thermal pressure can be neglected if [see Eq. (6)] $x_v \gg x_n \tau$, where x_v is the characteristic size of the void. Note that this inequality contains the parameter τ, which is usually small in existing experiments. For this limit, it is possible to use expression (3) for the ion drift velocity, and it is more convenient to introduce new dimensionless variables: $\tilde{E} \equiv E x_n$, which is the ratio of the work produced by the electric field on the ion-neutral collision mean free path to the electron temperature; $\tilde{x} \equiv x/x_n$, which is the distance in units of the ion-neutral collision mean free path; $D \equiv a/x_n\sqrt{\tau}$, which is the dust grain size (in units of the same mean free path) times $\sqrt{\tau}$; and $\tilde{x}_i \equiv x_i/x_n$, which is the ionization length in units of the same mean free path. The system of equations (5)–(8) in the void region in this case (and for this normalization) reduces for the following three equations for the three variables $\tilde{E}$, n_e, and Φ:

$$\frac{d\tilde{E}}{d\tilde{x}} = -\frac{1}{D^2}\left(n_e - \frac{\Phi\alpha_u}{\sqrt{1+\alpha_u\tilde{E}} - 1}\right), \tag{10}$$

$$\frac{dn_e}{d\tilde{x}} = -n_e \tilde{E}, \tag{11}$$

and

$$\frac{d\Phi}{d\tilde{x}} = \frac{n_e}{\tilde{x}_i}. \tag{12}$$

Another simplifying assumption could be applied for a quasineutral plasma, i.e., when $x_v \gg D$ (note that for x_v of the order of 1, we have simply $D \gg 1$ in this case) when we can neglect the left-hand side of Eq. (5). With this assumption, further simplifications can be done for any value of the ratio $x_i/\tau x_n$, i.e., for both the collision-dominated and collisionless voids. We find then without neglecting the ram pressure and the ion thermal pressure forces that

$$E = \frac{u}{1+\tau(1-2u^2)}\left[\frac{2\tau}{x_i} + \frac{1}{x_n}(2+\alpha_u u)\right]. \tag{13}$$

This example shows how other ion forces can modify the mobility and that the effective ionization length appears in the mobility coefficient together with the ion-neutral collision length. Equation (13) also shows that only for $\tau \ll 1$ and $x_n \ll x_i/\tau$ can one use expression (3) for the ion mobility. With expression (13), we have from Eqs. (5)–(8) only one equation,

$$\frac{du}{dx} = Eu + \frac{1}{x_i} = \frac{1}{x_i}\frac{1+\tau}{1+\tau(1-2u^2)} + \frac{u^2}{x_n}\frac{2+\alpha_u u}{1+\tau(1-2u^2)}. \tag{14}$$

This is an equation for the ion velocity u in the void region. For the case $\tau \ll 1$, it is possible (we remind the reader that the characteristic distances are redefined in this case as $\tilde{x} = x/x_n$ and $\tilde{x}_i = x_i/x_n$) to rewrite this equation as [cf. Eqs. (10)–(12)]

$$\frac{du}{d\tilde{x}} = \frac{1}{\tilde{x}_i} + u^2(2+\alpha_u u). \tag{15}$$

The electric field $\tilde{E} = Ex_n$ in this case is given by [see also Eq. (13)]

$$\tilde{E} = u(2+\alpha_u u). \tag{16}$$

IV. EQUATIONS IN THE DUST REGION

The momentum balance equation for plasma ions in the dust region contains the dust drag force F_{dr}, which is given by (see also [5])

$$F_{\mathrm{dr}} = \alpha_{\mathrm{dr}}\left(u, \frac{\tau}{z}\right) uzP \tag{17}$$

(we remind the reader that $z = Z_d e^2/a_d T_e$ and $P = N_d Z_d / N_{i0}$), where $\alpha_{\mathrm{dr}}(u, \tau/z)$ is the ion drag coefficient (which is the function of the ion drift velocity u and τ/z) and Z_d is the dimensionless dust charge in units of the electron charge. The balance equation for forces acting on dust takes into account only the electric and drag forces and is given by

$$E + \alpha_{\mathrm{dr}}\left(u, \frac{\tau}{z}\right) nuz = 0. \tag{18}$$

The drag coefficient α_{dr} is calculated taking into account both the capture force and the Coulomb scattering force [8]; its analytical expression through the error function erf(u) is given in the Appendix (note that the ‘‘collisionless’’ expression used in Ref. [5] can be obtained from that given in the Appendix in the limit $u \gg 1$).

The dust charges are determined by the charging equation

$$\exp(-z) = 2\sqrt{\pi}z \frac{n}{n_e} \frac{1}{\sqrt{\tau\mu}} \alpha_{\mathrm{ch}}\left(u, \frac{\tau}{z}\right), \tag{19}$$

where $\mu = m_i/m_e$ is the ion to electron mass ratio, and the capture coefficient $\alpha_{\mathrm{ch}}(u, \tau/z)$ in its exact form as a function of u and τ/z is given in the Appendix. For the case in which Eq. (19) is used at the boundary of the void (to find the value of the dust charge at the boundary), the boundary values for n and n_e can be substituted in Eq. (19). When the ion ram pressure and ion thermal pressure are neglected, we obtain the equation for dust charges at the void boundary,

$$n_e \exp(-z) = 2\sqrt{\pi}z \frac{\Phi}{\alpha_u}(\sqrt{1+\alpha_u E} - 1) \times \frac{1}{\sqrt{\tau\mu}} \alpha_{\mathrm{ch}}\left(\frac{\alpha_u}{\sqrt{1+\alpha_u E}-1}, \frac{\tau}{z}\right). \tag{20}$$

In the case of a quasineutral void, the dust charges can be found from a simpler relation,

$$\exp(-z) = 2\sqrt{\pi}z \frac{1}{\sqrt{\tau\mu}} \alpha_{\mathrm{ch}}\left(u, \frac{\tau}{z}\right). \tag{21}$$

The parameter P inside the dust region can be calculated explicitly as a function of other quantities in the dust region via the solution of Poisson’s equation in the dust region. The exact expression is given in the Appendix.

The size of the void can be found as a simultaneous solution of two boundary conditions: the continuity of the electric field and the dust charges at the surface of the void. The electric field from the void side is calculated numerically by solving the system of equations inside the void region. The electric field from the dust side is calculated using expression (18), for which it is necessary to know the dust charges at the void surface. The dust charges are calculated by the second boundary equation, which in the corresponding limits is given by Eq. (20) or Eq. (21), respectively.

V. NUMERICAL RESULTS

Here, we would like to mention that there are two natural dimensionless parameters of the system (describing the

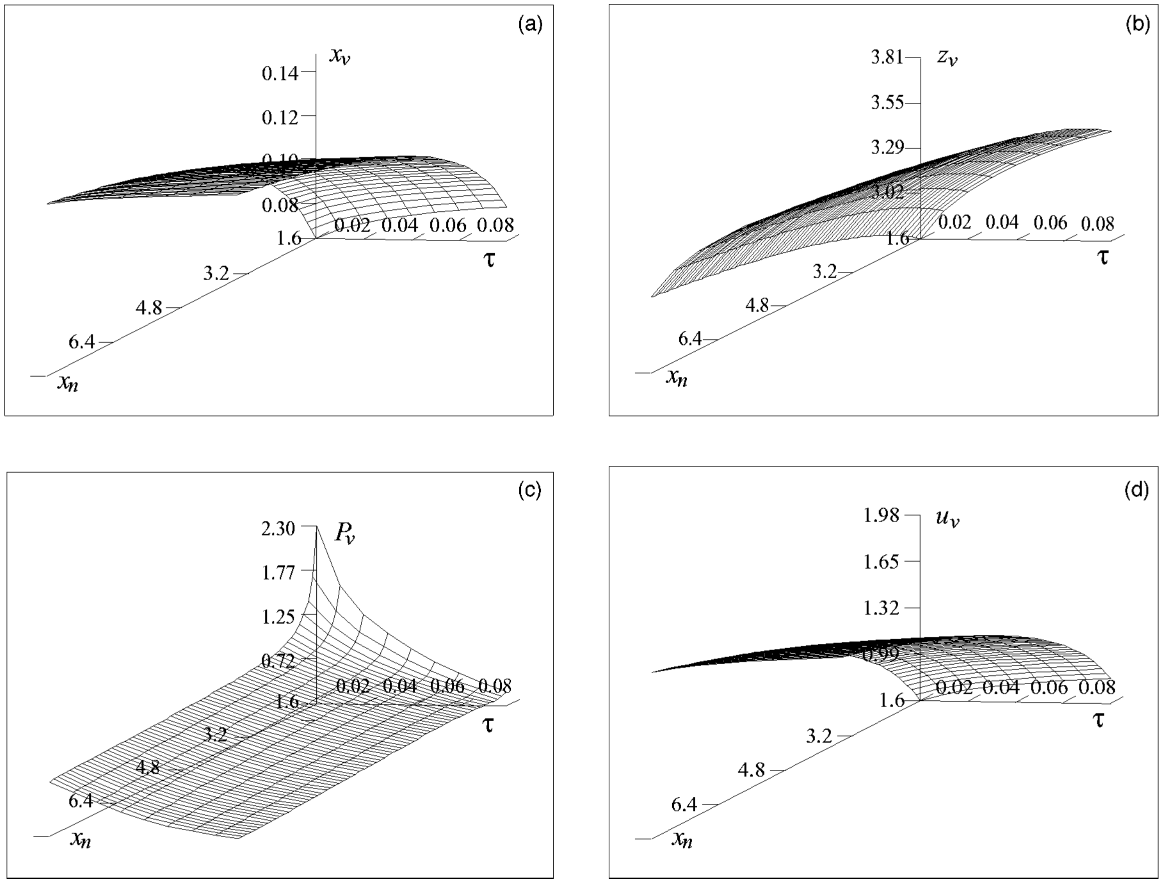

FIG. 1. Dependence of the void parameters on x_n and τ in the range $x_{n,\min}<x_n<10$ and $0.02<\tau<0.1$: (a) the surface plot for the dimensionless void size x_v, (b) the surface plot for the dimensionless dust charge at the void boundary z_v, (c) the surface plot for the dust density parameter jump P_v at the void boudary, and (d) the surface plot for the dimensionless ion velocity at the void boundary u_v. The results are obtained for quasineutral voids when $x_i=0.2$. The value of $x_{n,\min}$, where the void size is close to zero, is 1.6 for $\tau=0.02$; for larger τ, the minimum x_n decreases. The figure shows that the size of the void at lowest x_n increases with τ.

void), namely, D, which is the ratio of the void size to the electron Debye length, and x_i, which is the ratio of the ionization length to the ion-neutral collision mean free path. For the second parameter, the density of neutral species is canceled in the final expressions since both the ionization rate and the ion-neutral collision mean free path are proportional to the neutral density. Thus the parameter x_i does not depend on the neutral gas pressure and depends only on the ionization power. The parameter D is proportional to the neutral gas pressure. In numerical solution in the void region we will use either equations for the quasineutral void, which depend only on one parameter (normalized x_i) or for a void smaller than or of the order of the electron Debye length, the system of equations depending on both D and x_i.

In addition to the solution of the equations in the void region, it is necessary to satisfy the boundary conditions on the void surface, which additionally depend on the parameter x_n, namely the ratio of the ion-neutral collision mean free path to the ion-dust collisions mean free path times the parameter P. The value of the parameter P here is not arbitrary and corresponds to its value at the boundary of the void, which should be found as a solution of the boundary conditions. The value of P at the boundary of the void determines the jump of the dust density at the void surface. This jump can be obtained after dust charges at the boundary of the void are determined. The boundary conditions give two values: the dust charge at the void boundary and the size of the void. The electron and ion densities at the void surface and the value of ion drift velocity are found straightforwardly. The knowledge of these parameters is sufficient to find the value of the parameter P at the void boundary and thus the jump of dust density. There could be two possibilities for which the void does not exist: the absence of the solution for the boundary conditions and the negative value of the parameter P at the surface of the void.

The calculations are started in the center of the void $x=0$. The void is assumed to be planar and only 1D nonlinear equations are solved at $x>0$ (the void is also assumed to be symmetric with respect to $x=0$). At the center of the void, $x=0$, the electric field is zero, $E=0$, and the ion density is $n=1$ (in the normalization used). For the quasineutral voids, $n_e=n$ and the dimensionless electron density is also equal to 1. For nonquasineutral voids, it is necessary to use the proper asymptotics in the center of the void: $\Phi\rightarrow x/x_i, E\rightarrow 2x/x_i$ for $x\rightarrow 0$, which implies that $n_e\rightarrow 1-2D^2/x_i$ (this condition is the direct consequence of Poisson's equation). Since $n_e>0$ in the center, the nonquasineu-

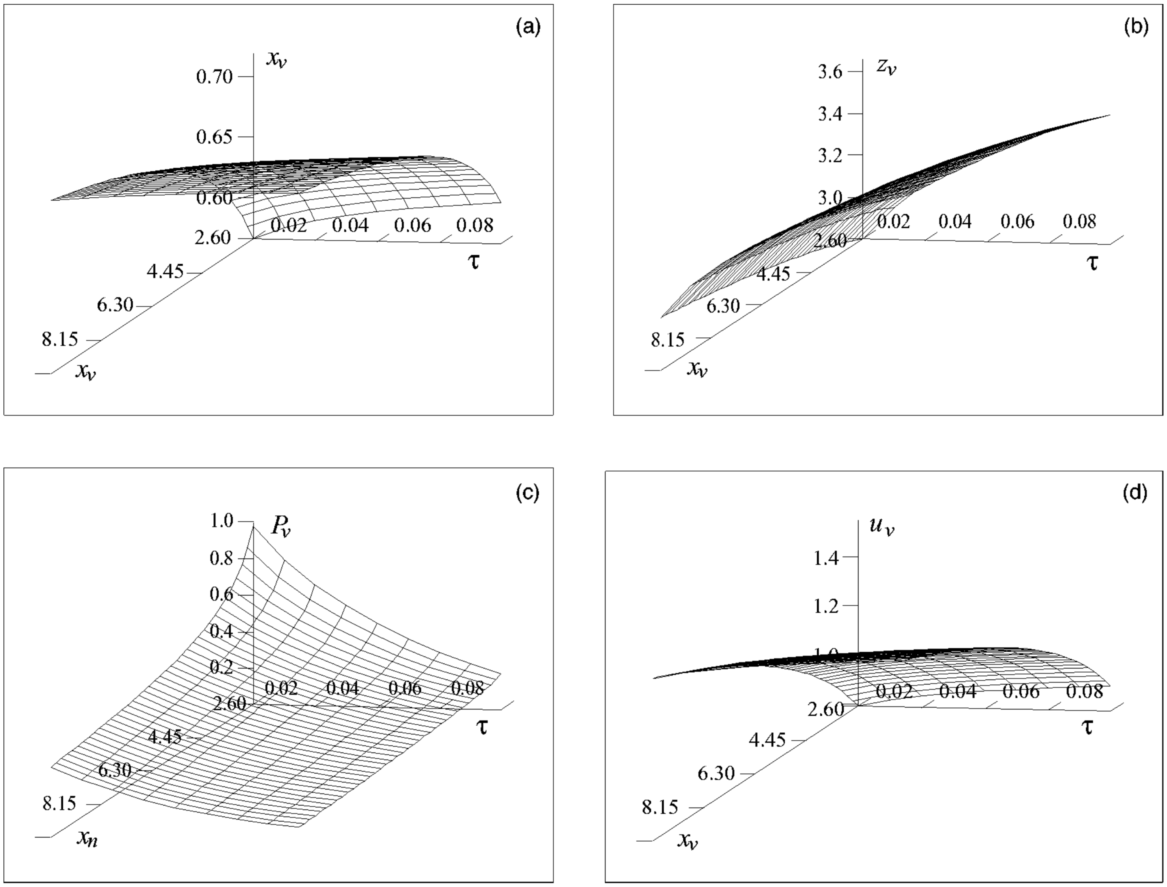

FIG. 2. The same as in Fig. 1 but for $x_i = 1$. The value of $x_{n,\min} = 2.6$ is larger than that in Fig. 1.

tral void can exist only for $x_i > 2D^2$. This condition is taken into account for the numerical solution of nonquasineutral dust voids.

For fixed D and x_i for nonquasineutral voids or for fixed x_i for quasineutral voids, the void exists if the parameter x_n exceeds some critical value. In some cases, the void exists within a certain range of the values of the parameter x_n between $x_{n,\min}$ and $x_{n,\max}$, and for other conditions the void can exist in several ranges of the parameter x_n. The boundary conditions depend also on the parameters $\tau = T_i/T_e$ and $a = a_d/d_i$. The calculations were performed for the parameter range most interesting for existing experiments: $0.02 < \tau < 0.1$ and $a = 0.1$. In principle, the boundary conditions can restrict the range of these parameters as well (for a void to exist). Thus an increase of τ can lower the threshold for x_n, as can be seen from the results given below. The results are presented for the parameter x_n exceeding the threshold for the smallest value of τ used. In this case, the results can be presented in the compact form as surface plots. All calculations are performed for argon plasma.

A. Numerical results for quasineutral voids

In the case of a quasineutral void, the calculations were performed for the the dimensionless parameter x_i (the ratio of the ionization length to the ion-neutral mean free path) being equal to 0.2, 1, and 5. The first case corresponds to the high ionization rate and the third case corresponds to the low ionization rate (high and low ionization power, respectively). Figures 1, 2, and 3 show the corresponding results for the allowed range of the void sizes for these three values of the parameter x_i. Figures 1(a), 2(a), and 3(a) show the size of the void x_v as a function of x_n and τ in the range $x_n > x_{n,\min}$ and $0.02 < \tau < 0.1$; Figs. 1(b), 2(b), and 3(b) show the dust charges z_v at the surface of the void in the same range; Figs. 1(c), 2(c), and 3(c) show the jump P_v of the parameter P at the surface of the void; and Figs. 1(d), 2(d), and 3(d) show the ion drift velocity u_v at the surface of the void. The calculations are performed until $x_n = 10$ (note that the latter is not related to the violation of the boundary conditions). For larger values of x_n the void can also exist, and this was checked by calculation of several points where x_n was larger than 10. For $x_n \approx 10$, the size of the void is already reaching an almost constant value and further calculations are not necessary. Note that the size of the void x_v is normalized to the mean free path of the ion-neutral collisions times the ion to electron temperature ratio τ, and the constant value of x_v in these units means that the void size is a fraction of this mean free path divided by τ, i.e., it can significantly exceed the ion-neutral mean free path. An increase of the mean free path in this limit leads to the increasing size of the void proportional to the mean free path. This asymptotic behavior is easy to interpret in a physical way: the size of the void is approximately equal to the ion-neutral collision mean

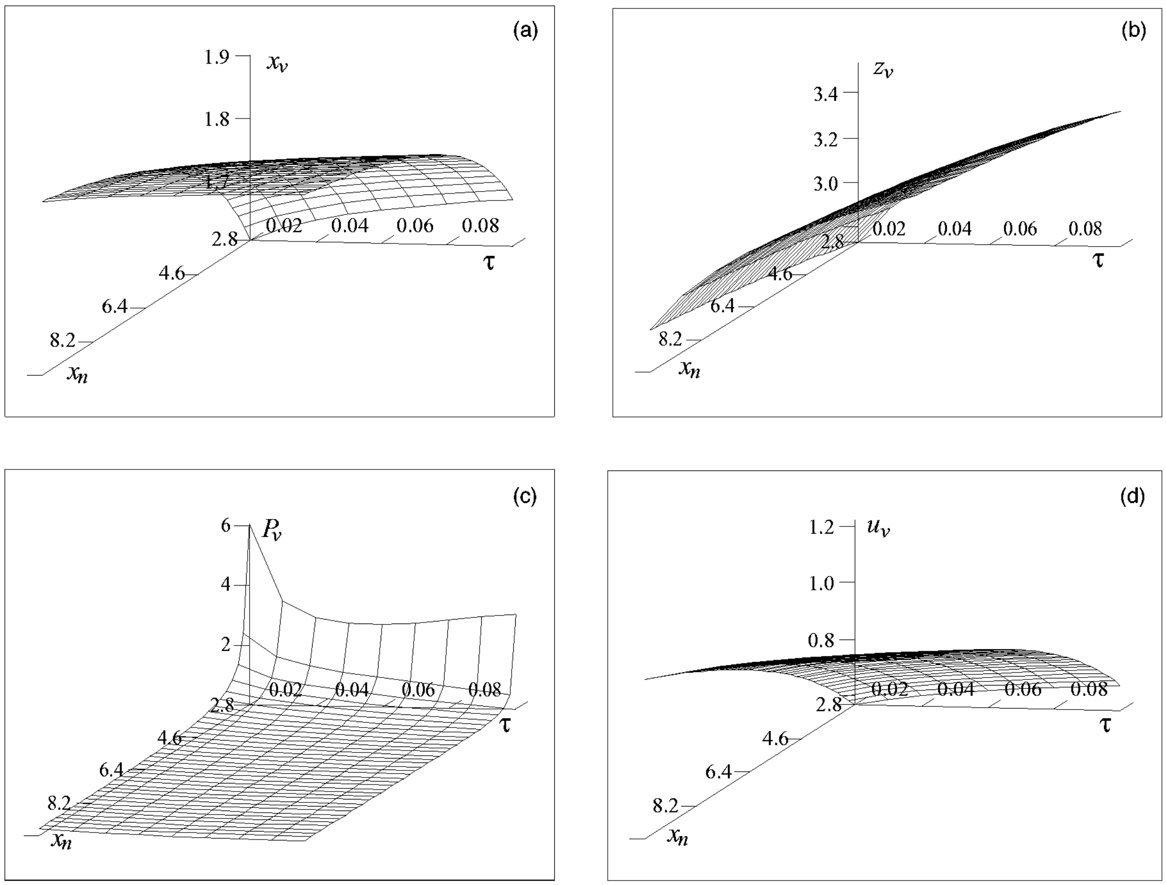

FIG. 3. The same as in Fig. 2 but for $x_i=5$; $x_{n,\min}=2.8$.

free path divided by τ. The voids larger than this size will not exist since the drag force will not increase with the size of the void (ions accelerated in the void will be stopped by the ion-neutral collisions). This behavior is found for all calculations performed for the quasineutral voids. Asymptotically, the constant determining what fraction of the mean free path (divided by τ) is the void size changes appreciably with the increase of x_i (the decrease of the ionization power), being 0.3 for $x_i=0.2$ and about 1 for $x_i=5$. The critical value of x_n for which the void starts to form is shown in Figs. 1, 2, and 3 as the minimum value of the x axis. The minimum value $x_{n,\min}$ increases gradually with x_i (with the decrease of the ionization power). This minimum value decreases with the increase of the temperature ratio τ. The jump of the parameter P at the void boundary, P_v, is the highest when the void size is close to $x_{n,\min}$. The ion drift velocities at the surface of the void are of the order of the ion thermal velocity, the minimum value being about 0.2 and the maximum value less than 2. With an increase of the ionization length x_i, the ion drift velocity increases.

B. Nonquasineutral voids

As soon as the size of the void becomes comparable to the electron Debye length, the condition of plasma quasineutrality is violated. The two series of numerical calculations were performed: one for $D=0.5$, $x_i=1$, which corresponds to the initial electron density $n_e=0.5$, and another for $D=3$, $x_i=40$, which corresponds to the initial electron density $n_e=0.45$. The case of $D=0.5$ corresponds to the marginal violation of plasma quasineutrality, while the case $D=3$ corresponds to the substantial violation of the quasineutrality. The results are shown in Figs. 4 and 5. The main features of the quasineutral void survive, including, specifically, the tendency to have asymptotically a void with size proportional to x_i, but the factor can be higher than 10. The minimum size $x_{n,\min}$ for $x_i=1$ further increases up to 2.7. The new phenomenon created by the absence of quasineutrality is the presence of two zones of possible values of x_n for a void to exist. The second zone not shown in Fig. 4 is for the relatively narrow range $1.1<x_n<1.2$. For the second set of numerical calculations for a large violation of quasineutrality, $D=3$ and $x_i=40$ (Fig. 5), it was revealed that the two zones of possible values of x_n determine the conditions for a void to exist. One of the zones is for the narrow range of x_n, namely $4.7<x_n<5.06$, and it is not shown in Fig. 5. Another zone that appears at rather low values of x_n, namely $0.69<x_n<3.4$, is presented in Fig. 5. For this zone, the value of ion drift velocity at the surface of the void becomes very large when the parameter x_n is approaching the largest possible values in the zone.

VI. DISCUSSION AND CONCLUSION

According to recent experiments [1–4], dust voids usually appear from a uniform dust cloud as a result of an instability

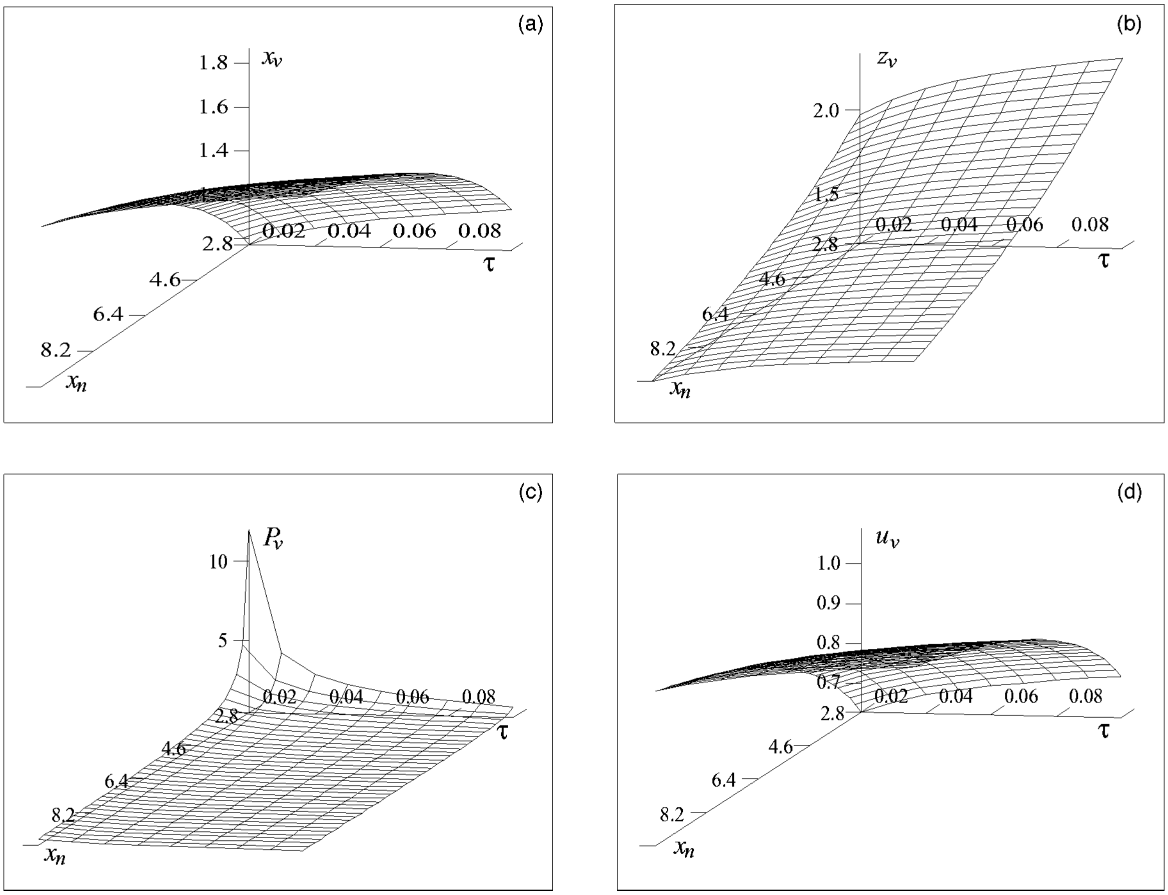

FIG. 4. The parameters of the nonquasineutral void for $x_i=1, D=0.5$ (compare with Fig. 2); $x_{n,\min}=2.8$.

associated with increased local ionization in spontaneously appearing depletions of the dust number density. In the region of reduced dust density, there are more electrons since fewer of them are absorbed by the dust. The higher electron density leads to higher ionization rate. This region of increased ionization will develop an increased positive space charge with respect to the surrounding medium. As a result, the outward ion flow develops. Thus there appear two forces acting on the negatively charged dust particles: an inward electrostatic force and an outward ion drag force. For a small particle size, the inward force will dominate and fill the dust density depletion so the fluctuation will disappear. However, if the particle exceeds a critical size, the outward ion drag force exceeds the electrostatic force. The region of reduced dust density will then expel more dust particles, and the fluctuation will grow. After sufficient growth, the mode becomes nonlinear and the growth saturates. One possible final state, as seen from this study, is a stable void.

The existence of a void requires a local source of ionization in a background plasma such as that in gas discharges used for plasma processing and dust-plasma crystal experiments, where the ionization is due to electron impact, although photoionization and other sources of ionization could have a similar effect. In the absence of local ionization, the dust cloud, a structure converse to the void, can appear. This situation can be applicable to astrophysical dust clouds [9] when, in the absence of local ionization, the ions must originate from a distant plasma source. Thus there is an inward ion flow that creates an inward ion drag force. This collapses the dust cloud, whose size also, like that of a void, can be regulated by the balance of two forces, but now they are the outward electrostatic force and the inward ion drag force.

In most of the existing experiments, the voids are either collision-dominated with respect to the ion-neutral collisions or marginal between the collision-dominated case and the collisionless case. Thus the present theory gives results that can be directly compared to experimental data. The results of experiments [2] can be qualitatively explained by the present theory. The size of the voids observed corresponds to the mean free path (with respect to the ion-neutral collisions) divided by the ion/electron temperature ratio, i.e., x_n/τ in our dimensionless units, which corresponds to the experiments [2–4]. An increase of the size of the void with an increase of the ionization power is observed in coincidence with the theoretical results of the present paper. The larger the size of the dust particles, the smaller is the mean free path in ion-dust collisions d_i^2/a_d, thus for fixed pressure the value of x_n (the dimensional ionization distance is normalized by d_i^2/a_d) increases and a comparison can be made with the mentioned asymptotical behavior of dust void size with the mean free path. The validity of the results are confirmed since the value of the ion drift velocity at the surface of the void is not larger than 2 when the use of the dependence of

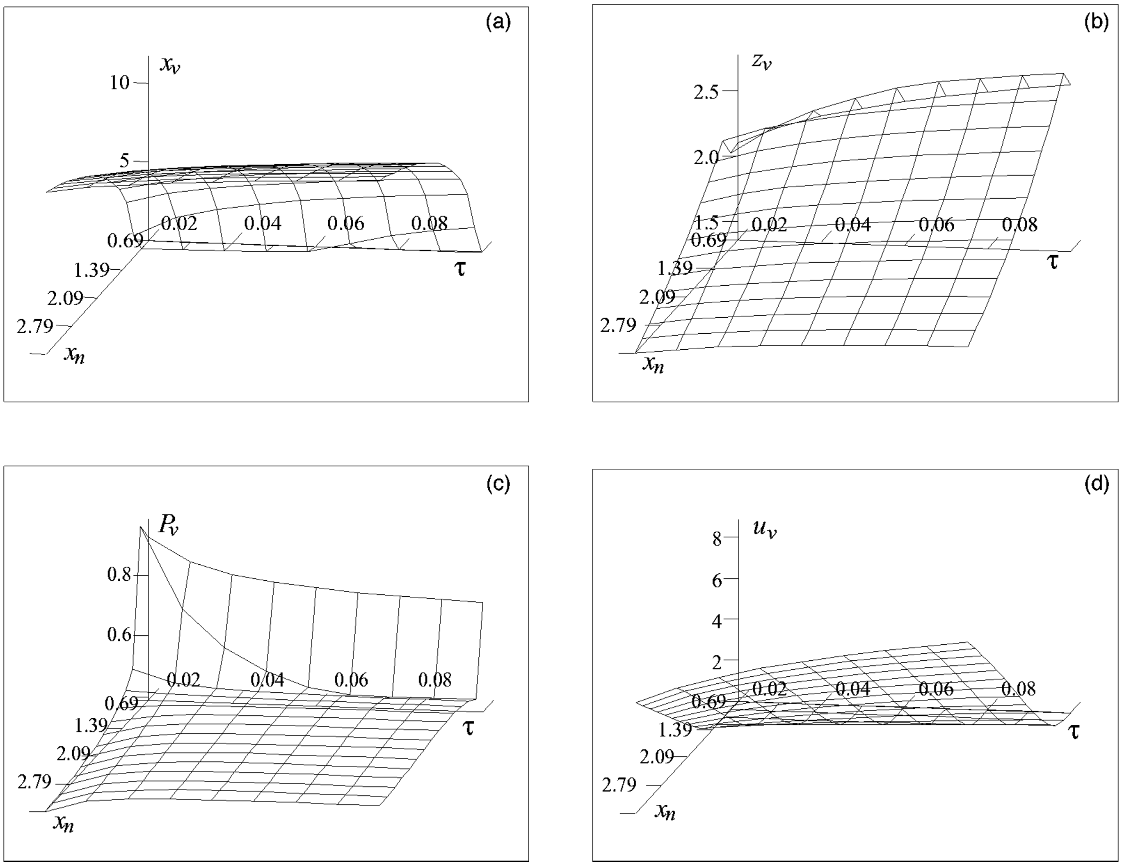

FIG. 5. The parameters of the nonquasineutral void for $x_i = 40, D = 3$ (compare with Fig. 4); $x_{n,\min} = 0.69$.

ion mobility on the drift velocity used in the model is known to be experimentally verified.

We note that a ''collisionless'' void [5] can be sustained by the ion drag force when the ions have a velocity greater than the ion thermal speed, $u \gg 1$ in our nondimensional units. That is because the ion drag force increases with u for $u < 1$ and then it decreases with u for $u > 1$; at still higher ion speeds, where the Mach number M is greater than unity, $M > 1$, there is a third regime dominated by the ion collection force by the dust where the ion drag force once again increases with u; see Fig. 3 of Ref. [5]. We recall that in the collisionless void model, as ions flowed outward from the void center, the ion velocity was greater than the ion thermal speed, $u > 1$, and u increased with the distance from the void center. That meant that they were in the regime in which the ion drag force diminished with distance from the void center. At a sufficient distance from the void center, u becomes sufficiently high that the ion drag force is no longer bigger than the Coulomb force in the opposite direction, and then there can be dust particles present.

In the colllisional case, see Figs. 6 and 7 of the present paper, it appears that $u \leqslant 1$ almost everywhere in the void. That means that the ion drag force is basically in the regime in which the force increases with u, which is different from the collisionless case studied in Ref. [5]. In particular, in the collisional case, the electric field and the ion drag force both increase with the ion speed. However, the faster increase of the electric field E can lead to the force balance, especially in the case in which $u \sim 1$, when the further increase (with the distance off the center of the void) of the ion drag force actually stops. Note that the dimensionless ion speed is typically of order unity at the void edge. On the other hand, the charge of a dust particle enters also the force balance equation, and is also a function of the distance. The result is a

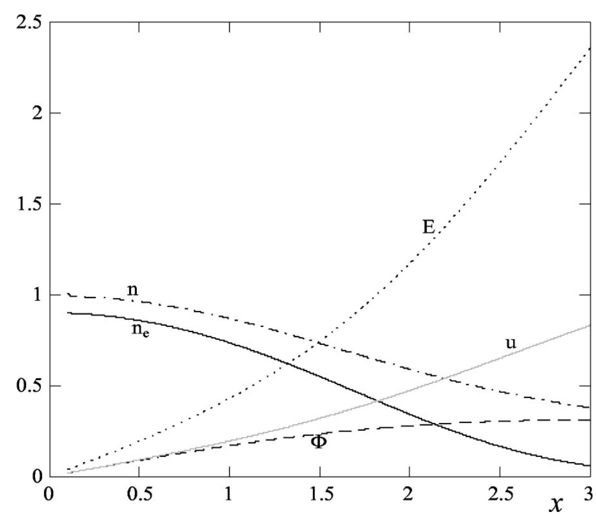

FIG. 6. An example of distributions of the electron and ion densities, electric field, ion flux, and ion drift velocities in the void region for the case $x_i = 5, D = 0.5$ The size of the void (where the lines should finish) is not determined.

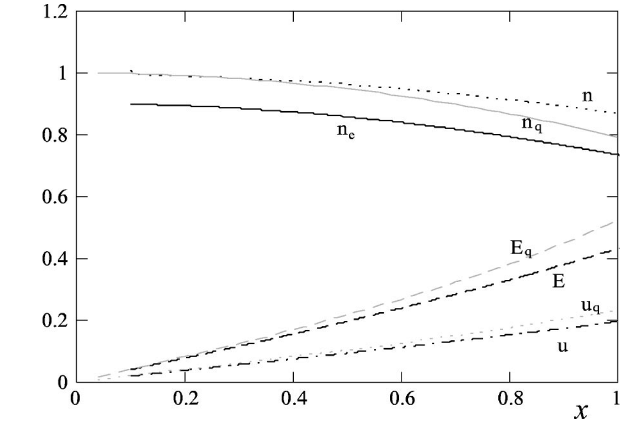

FIG. 7. Comparison of the distribution of void parameters when the quasineutral assumption is made (the parameters marked by the superscript q) with the distribution of the void parameters when the quasineutral assumption is not made for the case of marginal quasineutrality conditions. The value $x_i = 5$ is the same in comparison while the value of D is $D = 0.5$.

complex interplay of a number of functions of the distance from the center of the void (the electric field, dust charge, speed of the ion flow, etc.); see Fig. 6.

In our calculations, we neglected the (possible) influence of the thermophoretic force associated with temperature gradients. The gradient of the neutral temperature in the void region is produced by the flowing ions colliding with the neutral species. The drift velocity of the ions increases towards the boundary of the void. However, for the thermophoretic force to be comparable with the other forces (e.g., the ion drag force), we have taken into account in our calculations that it is necessary for the drift velocity to be of order of the Bohm velocity, i.e., u of the order of $(m_i/m_e)^{1/2}$, which we can hardly observe in these collision-dominated voids, and the assumption of the neglect of the thermophoretic force is justified. We finally note that the present theory does not provide the initial transition stages of the void formation, but rather demonstrates the existence of stationary solutions in the case of bordering void-dust regions. Instabilities leading to the void formation should be studied separately.

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APPENDIX: GENERAL EXPRESSIONS IN THE DUST REGION

The plasma capture (charging) coefficient $\alpha_{\rm ch}$ and the plasma drag coefficients $\alpha_{\rm dr}$ have the following dependence on $t = \tau/z$ and on the ion drift velocity u:

$$\alpha_{\rm ch} = \frac{\mathrm{erf}(u)}{8u}(2+t+3u^2 t) - \frac{t\exp(-u^2)}{4\sqrt{\pi}}, \tag{A1}$$

$$\alpha_{\rm dr} = \frac{\mathrm{erf}(u)}{8u^3}\left[t(-1+4u^2+4u^4)+2t(-1+2u^2)+4\ln\left(\frac{1}{a}\right)\right] + \frac{\exp(-u^2)}{4\sqrt{\pi}u^2}\left[t[t(1+2u^2)+2]-4\ln\left(\frac{1}{a}\right)\right]. \tag{A2}$$

The solution of the Poisson equation in the dust region gives the analytic expression for the parameter P,

$$P = \frac{\left[(n-n_e)\frac{\tau}{a^2}+u^2 n\alpha_{\rm dr}\frac{z}{R}\frac{\partial(\alpha_{\rm dr}z)}{\partial z} - z n_e \alpha_{\rm dr}\alpha_u\right](1-2u^2)+A}{\left(\frac{\tau}{a^2}-zn\alpha_{\rm dr}\alpha_{ch}\right)(1-2u^2)-B}, \tag{A3}$$

where

$$R = 1 + \frac{\mathrm{erf}(u)}{4uz\alpha_{\rm ch}}, \tag{A4}$$

$$A = \frac{nu}{R}\left(\frac{\partial(z\alpha_{\rm dr})}{\partial z}\right)\left\{nuz\alpha_{\rm dr}+2u\alpha_{\rm ch}\frac{n_e}{n}-\frac{1}{x_n}u(2+\alpha_u u)-\frac{1}{\alpha_{\rm ch}}\left(\frac{\partial\alpha_{\rm ch}}{\partial u}\right)\left[u^2 nz\alpha_{\rm dr}-\alpha_{\rm ch}\frac{n_e}{n}-u^2\frac{1}{x_n}(2+\alpha_u u)\right]\right\} + nuz\left(\frac{\partial\alpha_{\rm dr}}{\partial u}\right)\left[\frac{u^2 zn\alpha_{\rm dr}}{\tau}-u^2\frac{1}{x_n}(2+\alpha_u u)-\alpha_{\rm ch}\frac{n_e}{n}\right]. \tag{A5}$$

$$B = \frac{nu}{R}\left(\frac{\partial(z\alpha_{\rm dr})}{\partial z}\right)\left[u\left(2\alpha_{\rm ch}-\frac{z}{\tau}\alpha_{\rm dr}\right)-\frac{\partial\alpha_{\rm ch}}{\partial u}+\frac{u^2 z\alpha_{\rm dr}}{\tau\alpha_{\rm ch}}\frac{\partial\alpha_{\rm ch}}{\partial u}\right]+nuz\left(\alpha_{\rm ch}-u^2\alpha_{\rm dr}\frac{z}{\tau}\right)\frac{\partial\alpha_{\rm dr}}{\partial u}. \tag{A6}$$

The value of parameter P at the surface of the void can be found by substituting in Eq. (A6) the densities of electrons and ions and the ion drift velocity from the results obtained numerically in the void region at the position at the void surface. The parameter z at the void surface is found directly by solving the boundary conditions.

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